

# Alternative Marginal Fluctuations in Form of Instantons on D2/M2-Brane Theory

M. Naghdi \*

*Department of Physics, Faculty of Basic Sciences,  
University of Ilam, Ilam, West of Iran.*

February 20, 2013

## Abstract

We introduce some (anti) M/D-branes through turning on some 4-form field-strengths for eleven- and ten-dimensional supergravity theories over  $AdS_4 \times M_{7|6}$  spaces, where we use  $S^7/Z_k$  and  $CP^3$  for the internal spaces. Indeed when we add M2/D2-branes on the same directions with near horizon branes of Aharony, Bergman, Jafferis and Maldacena model, all symmetries and supersymmetries are preserved trivially. In the case we gain a localized object just in the horizon. This normalizable bulk massless scalar mode is a singlet of  $SU(4) \times U(1)$  and  $SO(8)$  and also agrees to an exactly marginal boundary operator of the conformal dimension of  $\Delta_+ = 3$ . However, after performing a special conformal transformation we see the solution is localized in all Euclidean  $AdS_4$  space and is attributable to the thought anti M2/D2-branes adding also necessary to ensure there is no back-reaction. The resultant theory now breaks all  $\mathcal{N} = 8, 6$  supersymmetries to  $\mathcal{N} = 0$ , while other symmetries are so preserved. The dual boundary operator then set up from a skew-whiffing of the representations  $\mathbf{8}_s$  and  $\mathbf{8}_v$  for supercharges and scalars respectively while the fermions remain fixed in  $\mathbf{8}_c$  of the original theory. Besides, we also address another alternate bulk to boundary matching procedure through one of the gauge fields of the full  $U(N)_k \times U(N)_{-k}$  gauge group in same lines with similar situation faced in  $AdS_5/CFT_4$  correspondence. The latter approach covers the already met obstacle of the bulk-boundary matching procedure for  $k = 1, 2$  as well.

---

\*E-Mail: m.naghdi@mail.ilam.ac.ir

# 1 Introduction

In some recent studies [1], [2], [3], [4] we explored more on the vacua of the Aharony, Bergman, Jafferis and Maldacena (hereafter ABJM) model [5] as by now the best well-conjectured model of  $AdS_4/CFT_3$  duality or M2/D2-brane theory. The main class of solutions we have been looking for are Solitons and Instantons. The instantons we look for defines as solutions to the classical equations of motions (EOM hereafter) of the Euclidean space with nonzero finite actions so there are some saddle points contributing to the phase integral with sharing non-perturbative corrections to the main action. The manner we use is turning on some fluxes via the form field strengths, while we try to keep the original ABJM background fields and geometry [5] unaffected. The arisen effects may be considered as coming because of giving some dynamics to the original branes or as adding or embedding new branes.

We first found a kind of M-brane instanton [1]. Indeed, by making use of some ansatzs for 4-form field strength and EOM's of eleven-dimensional (11d hereafter) supergravity over  $AdS_4 \times S^7$  when  $S^7$  is considered as a  $S^1$  fiber on  $CP^3$ , we arrived in a localized object in bulk of  $AdS_4$ . The corresponding *irrelevant* boundary operators of the conformal dimension  $\Delta_{\mp} = 1, 2$  with the bulk tachyonic fields were surveyed. Then, to adjust the bulk and boundary theories we were forced to exchange the representations  $\mathbf{8}_s$  and  $\mathbf{8}_c$  for supercharges and spinors of the ABJM model, respectively. The resultant theory then was for anti M2-branes. With turning on the singlet fermi field next to just the  $U(1)$ 's parts of the full gauge group, the fitted dual boundary solution with finite action was also obtained.

The second exact solution was for  $U(1)$  instantons [2]. Turning on some gravity field induced some magmatic fluctuations on the boundary theory. For these massless gauge fields in the bulk of  $AdS_4$ , dual boundary operators of the dimensions of  $\Delta_{\mp} = 1, 2$  were now constructed just from  $U(1) \times U(1)$  gauge fields. That solution was indeed a D0-instanton on the background D2-branes. Electric-magnetic duality in the case and adjusting bulk-boundary solutions were also referenced, while another related viewpoint on the issue next to an uplift of the solution to 11d was performed in [3].

Next, we arrived in a new fully localized solution in Euclidean  $AdS_4$  ( $EAdS_4$ ) [4]. That is a pseudo-scalar coming from anti D4(M5)-branes their world-volumes are warping on five (six) directions of  $CP^3$  ( $S^7/Z_k$ ). Actually, the field strength to which the brane couples is a 6(7)-form written in terms of the nontrivial one-form  $\omega$  on  $CP^3$ . The gravity ansatz is  $SU(4) \times U(1)$ -invariant and the massless pseudo-scalar therein exists in already known spectrum of 10d (11d) supergravity over the associated space when, of course,  $S^7/Z_k$  is considered as a  $U(1)$  fiber-bundle on  $CP^3$  [6]. Meanwhile, the solution breaks supersymmetries and supercharges have to be in  $\mathbf{8}_c$  or  $\mathbf{8}_v$  in contrast to ABJM. This means the resultant theory is again for anti-membranes because of adding the supersymmetry breaking anti-branes. To adjust boundary to bulk we swap the representations  $\mathbf{8}_s$  and  $\mathbf{8}_c$  of supercharge and spinors of the ABJM model similar to [1]. The boundary *marginal* operator of the conformal dimension  $\Delta_+ = 3$ , associated with normalizable bulk mode, have same structure as the terms in  $SU(4) \times U(1)$ -invariant action of ABJM first presented in [5] and [7], explicitly. By analyzing the behavior of bulk mode near to the boundary and making use of the bulk-boundary duality rules [8], [9] we

determined the matching field theory solution and comment on other related issues as well. It is mentionable that to arrive in a clear bulk solution we needed to turn on some scalars, fermions alongside  $U(1)$  parts of the completed gauge group. There we also discuss on the uplift of ansatz to 11-dimension and argue that our solution is not at least valid for  $k = 1, 2$ , where the R-symmetry is enhanced to  $SO(8)$ . Another interesting hint was that we could use the boundary gauge fields to find the associated solution to the bulk one. With that, we could say that our instanton solution may be the best counterpart for 10d type IIB supergravity one over  $AdS_5 \times S^5$  versus 4d  $\mathcal{N} = 4$   $SU(N)$  Yang-Mills theory [10], [11]. In the current note we continue the lines of study forward.

The instanton solution in the bulk here is almost of same structure with [4] except some subtle points. The first one is while the 6-form and associated bulk equation in former case was not invariant under conformal transformation, the 4-form ansatz here in 10d and 11d supergravities over  $AdS_4 \times CP^3$  and  $S^7/Z_k$  respectively are conformal invariant. Indeed we find some massless scalar solutions whose conformal transformation or their skew-whiffing go to the bulk solution in [4]. The basic solutions here preserve all supersymmetries while the associated D2/M2-branes are added on same directions as the original ones in near horizon limit of ABJM. The conformably transformed or skew-whiffed versions break all supersymmetry as expected, except when the internal space is  $S^7$ . Other symmetries are simply preserved and the associated boundary operator is then *exactly marginal*. An important point is that to not faced with back-reactions because of the added effects, our special conformal transformation (or the special skew-whiffing) is essential.

On the other hand, we know the massless scalars are in representation  $\mathbf{35}_v \rightarrow \mathbf{15}_0 \oplus \mathbf{10}_2 \oplus \bar{\mathbf{10}}_{-2}$  of  $SO(8) \rightarrow SU(4) \times U(1)$  in the original ABJM presentation. So the uncharged scalars sit in  $\mathbf{15}_0$ . Nevertheless, one may build the needed  $SU(4)$ -invariant  $U(1)$ -natural dimension-three operators from the associated scalars in  $\mathbf{8}_v = \mathbf{4}_1 \oplus \bar{\mathbf{4}}_{-1}$  to adjust with the bulk mode. Still the main and of course interesting solution is the special skew-whiffed one. For the latter case, we must again exchange the representation  $\mathbf{8}_s$  with  $\mathbf{8}_c$  or  $\mathbf{8}_v$  in ABJM. The best one here will be  $\mathbf{8}_v \rightarrow \mathbf{8}_s = \mathbf{1}_2 \oplus \mathbf{1}_{-2} \oplus \mathbf{6}_0$  while the fermions remain in  $\mathbf{8}_c = \mathbf{4}_{-1} \oplus \bar{\mathbf{4}}_1$  unchanged. Upon this, the scalars now sit in  $\mathbf{35}_v \rightarrow \mathbf{35}_s = \mathbf{1}_0 \oplus \bar{\mathbf{1}}_4 \oplus \mathbf{1}_{-4} \oplus \bar{\mathbf{6}}_2 \oplus \mathbf{6}_{-2} \oplus \mathbf{20}_0$ . In this case we lead again to anti D2/M2-brane theory by the new effects. Analyzing the bulk solution near to the boundary in similar line with [4] next to our knowledge of general forms for the marginal operators, guide us to find some boundary fitted solutions. Indeed, we use a single scalar instead of the single fermion of [1], [3] next to setting fermions and gauge fields to zero to build the field theory solution with finite action and matching with the bulk facts.

Yet another alternative and even more proper matching elements on the boundary could be gauge fields. Actually, we will use some gauge fields of  $SU(N) \times SU(N)$  gauge group to build the respective marginal operator and solution to match with bulk. The operator is composed of elements of the Chern-Simon terms common in M2/D2-branes theories. For simplicity and based on some arguments, of course, we just consider one of  $U(N)$ 's and so concentrate on the famous  $SU(2)$  gauge group. Next steps are similar and based on matching the same type bulk instanton solution in  $AdS_5$  with Yang-Mills instantons on the boundary of 4d  $SU(N)$

SYM theory [11], [12], [13]. As we see this solution is now to match with both common bulk solutions here and in [4].

The rest of paper organizes as follows. In Section 2, we deal with the gravity side of study including the needed material of involved supergravity theories, ansatzs for form-fields both in 10d and 11d, associated supergravity spectrums, clear solutions, charges, actions and some related gravitational discussions. In Section 3, field theory side is addressed. There we continue with matching bulk solution to the boundary by correspondence and find plain boundary solutions; while in the separate Section 4, we study the boundary solutions because of turning on the Yang-Mills gauge fields and how to adjust with the bulk solution. Section 5 includes summary and some related issues may not addressed in other parts.

## 2 On Gravity Side Aspects

### 2.1 Background Geometries and Fields of M2/D2-Branes, Actions and Equations of Motion

We first review the needed materials for D2/M2-brane supergravity theories that is mainly from ABJM [5]. One always starts from M-theory over  $AdS_4 \times S^7$  with  $\acute{N}$  unit of 4-form flux as:

$$ds_M^2 = \frac{R^2}{4} ds_{AdS_4}^2 + R^2 ds_{S^7}^2 \quad (2.1)$$

$$G_4^{(0)} \sim \acute{N} \mathcal{E}_{AdS_4} \quad (2.2)$$

where  $R$ ,  $\acute{N}$  and  $\mathcal{E}_{AdS_4}$  are the curvature radius of 11d target-space, the initial number of flux quanta and unit volume-form of  $AdS_4$ , respectively. For  $AdS_4$  metric in Poincare upper-half plane coordinate with Euclidean signature we use

$$ds_{EAdS_4}^2 = \frac{R^2}{4u^2} (du^2 + dx_i dx_i), \quad i = 1, 2, 3 \quad (2.3)$$

where  $R = R_7 = 2R_{AdS} = 2L$ . For  $S^7$ , when considered as a  $S^1$  fiber-bundle on  $CP^3$ , we write

$$ds_{S^7}^2 = ds_{CP^3}^2 + (d\phi + \omega)^2, \quad (2.4)$$

where  $\phi$  is the fiber coordinate with a period of  $2\pi$  and  $\omega$  is a topologically nontrivial one form related to the Kähler form  $J$  on  $CP^3$  (indeed  $J = d\omega$ ) and dual to the Reeb killing vector of  $\partial_{\phi}$ . In the last metric, the original isometry symmetry of  $S^7$  breaks down as  $SO(8) \rightarrow SU(4) \times U(1)$ , where  $SU(4)$  is the isometry of  $CP^3$ . After taking the orbifold  $Z_k$  of  $C^4$  coordinates transverse to M2-brane world-volume, these 8 scalars convert as  $X_I \rightarrow e^{i2\pi/k} X_I$  with  $(I = 1, 2, 3, 4)$  and  $\phi \rightarrow \phi/k$  and of course M-theory is now over  $AdS_4 \times S^7/Z_k$ . When  $k$  becomes large (indeed  $k \rightarrow \infty$ ), the M-theory circle becomes small and a better description is 10-dimensional type IIA string theory over  $AdS_4 \times CP^3$  with  $\acute{N} = kN$  unit of 4-form flux

on the quotient space:

$$ds_{ABJM(IIA)}^2 = \tilde{R}^2 (ds_{AdS_4}^2 + 4ds_{CP^3}^2), \quad \tilde{R}^2 = \frac{R^3}{4k} = \pi\sqrt{2\lambda} \quad (2.5)$$

where  $\lambda \equiv N/k$  is the 't Hooft coupling of boundary theory and the approximation is valid when  $\lambda \gg 1$  and also  $k^5 \gg N$ . We now have  $N$  unit of 4-form flux on  $AdS_4$  and  $k$  unit of 2-form flux on 2-cycle  $CP^1 \subset CP^3$  as

$$F_2^{(0)} = dA_1^{(0)} = kJ, \quad F_4^{(0)} = dA_3^{(0)} = \frac{3}{8}R^3\mathcal{E}_4, \quad H_3 = dB_2 = 0, \quad e^{2\phi} = \frac{R^3}{k^3} \quad (2.6)$$

with  $\mathcal{E}_4$  as the unit-volume form on  $AdS_4$  and  $B_2$  as NSNS 2-form field of type II theories.

We now comment on the actions from which above backgrounds arise and our solutions have to satisfy the equations of motion as well. First we concentrate on 10d type IIA supergravity action and equations we working with mainly, while 11d supergravity discussions come in its own place briefly. The 10d type IIA supergravity in string frame always reads

$$S_{IIA} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{g} e^{-2\phi} R + \frac{1}{2\kappa^2} \int \left[ e^{-2\phi} (4d\phi \wedge *d\phi - \frac{1}{2}H_3 \wedge *H_3) - \frac{1}{2}F_2 \wedge *F_2 - \frac{1}{2}\tilde{F}_4 \wedge *\tilde{F}_4 - \frac{1}{2}B_2 \wedge F_4 \wedge F_4 \right] \quad (2.7)$$

where  $\tilde{F}_4 = dA_3 - A_1 \wedge H_3$  and the Hodge-star operation is with respect to the full 10d metric. Among the EOM's for the form-fields, metric and dilaton, the dilation equation is satisfied trivially for ABJM as the dilation take a constant value,  $H_3 = 0$  and that Ricci-scalar  $R$  vanish for the involved geometries. The formic equations that new solutions should satisfy, with  $H_3 = 0$ , are as well

$$dF_p = 0, \quad d * F_p = 0, \quad (2.8)$$

$$d(e^{-2\phi} * H_3) = -F_2 \wedge *\tilde{F}_4 + \frac{1}{2}\tilde{F}_4 \wedge \tilde{F}_4 \quad (2.9)$$

where  $p = 2, 4$ . The Einstein equations

$$R_{MN} - \frac{1}{2}g_{MN}R = -8T_{MN}^\phi + T_{MN}^{H_3} + e^{+2\phi}T_{MN}^{F_2} + e^{+2\phi}T_{MN}^{\tilde{F}_4} \quad (2.10)$$

where the capital indices  $M, N, \dots$  are for the 10d space-time directions, satisfy with the ABJM background. But we should note that as long as we don't interest in changing of the background geometries, we should adjust the added fields so energy-momentum tensors on the right hand sides (RHS) vanish because of the extra effects. Nonetheless, as we argued in the previous studies [2], [4] even though there may be some small back-reactions on the original background, one could simply ignore in a probe-brane limit or because of holographic renormalization [14]. However, we see here that back-reactions are small when we add few branes in parallel directions with the original ones or vanish when we add a skew-whiffed version of the former branes, i.e. some special anti-branes.

## 2.2 Ten-Dimensional Ansatzs and Solutions

The ansatz we consider for 4-form field-strength of 10d type IIA supergravity over  $AdS_4 \times M_6$ , with  $M_6$  as a common 6d internal space, are

$$A_3^{(1)} = (f_1 A_3^{(0)}) \Rightarrow F_4^{(1)} = df_1 \wedge A_3^{(0)} + f_1 F_4^{(0)} \quad (2.11)$$

$$F_4^{(2)} = df_2 \wedge dx \wedge dy \wedge dz \quad (2.12)$$

$$F_4^{(3)} = d\left(\frac{1}{f_3}\right) \wedge dx \wedge dy \wedge dz \quad (2.13)$$

where  $f_1, f_2, f_3$  are some scalar functions in terms of bulk coordinates. The identity in (2.8) satisfies trivially and from satisfying  $d * F_4 = 0$  we obtain the following differential equations

$$\frac{d^2 f_1(u)}{du^2} - \frac{2}{u} \frac{df_1(u)}{du} \equiv L_1 f_1(u) = 0 \quad (2.14)$$

$$\frac{d^2 f_2(u)}{du^2} + \frac{4}{u} \frac{df_2(u)}{du} \equiv L_2 f_2(u) = 0 \quad (2.15)$$

$$L_2 f_3(u) - \frac{2}{f_3(u)} \left( \frac{df_3(u)}{du} \right)^2 \equiv L_3 f_3(u) = 0 \quad (2.16)$$

respectively. The solutions so are

$$f_1(u) = c_1 + c_2 u^3 \quad (2.17)$$

$$f_2(u) = c_3 + \frac{c_4}{u^3} \quad (2.18)$$

$$f_3(u) = \frac{c_5 u^3}{c_6 - 3c_7 u^3} \quad (2.19)$$

where  $c_1, c_2, \dots$  are some constants related to the object charges we encounter more. Now, we note the operators  $L_1, L_2, L_3$  are invariant under the following conformal transformation

$$x_{\dot{\mu}} \leftrightarrow \frac{x_{\dot{\mu}}}{u^2 + r^2} \quad (2.20)$$

where  $\dot{\mu}, \dot{\nu}, \dots$  we use for four  $AdS_4$  directions and  $r = \sqrt{x_i x^i}$ . The conformal transformation maps a point at infinity to another at origin meanwhile interchanges the boundary conditions. In other words, the compact space is now  $S^3 \times CP^3(S^7/Z_k)$  with a normal vector reversed and the sign of 4-form fluxes changes because of the sign changing of  $\mathcal{E}_4$  by the map. In our case it transforms instantons to anti-instantons actually. We come back to this point later. On the other hand, the metrics (2.1) and (2.5) and also 4-form fields are invariant under this transformation. Therefore, as  $f(u)$  is a solution also is  $f(\frac{u}{u^2+r^2})$ . So, from (2.17) we have

$$f_1^t(u) \equiv f^t(u, \vec{u}; 0, \vec{u}_0) = c_1 + \frac{c_2 u^3}{[(\vec{u} - \vec{u}_0)^2 + u^2]^3} \quad (2.21)$$

this is indeed the boundary to bulk propagator. With  $c_7 = 0$ , (2.19) leads to same structure while we put (2.18) aside for now in there is no new thing and refer to [15] for a similar study in type IIB theory. This conformally transformed solution is same as that already met in [4] as a skew-whiffed solution attributable to anti D/M-instantons. This solution is also of similar type with D-instanton in type IIB supergravity over  $AdS_5 \times S^5$  [11], [12], [13].

In order to see what happen really for our ansatzs based on solutions and under the transformation (2.20), we could write

$$F_4^{(1)} = \frac{3}{8}R^3(f_1(u) - \frac{u}{3}\dot{f}_1(u)) \mathcal{E}_4 \Rightarrow F_4^{(1a,1b)} = \pm \frac{3}{8}R^3c_1\mathcal{E}_4 \quad (2.22)$$

$$F_4^{(2)} = -u^4\dot{f}_2(u) \mathcal{E}_4 \Rightarrow F_4^{(2a,2b)} = \pm 3c_4\mathcal{E}_4 \quad (2.23)$$

$$F_4^{(3)} = u^4 \frac{\dot{f}_2(u)}{f_2(u)^2} \mathcal{E}_4 \Rightarrow F_4^{(3a,3b)} = \pm \frac{3}{c_8}\mathcal{E}_4 \quad (2.24)$$

where  $\dot{\phantom{x}}$  on  $f$ 's stands for the first derivative with respect to  $u$ ,  $c_8 = c_5/c_6$  and the upper sign  $+$  is for the original solutions while the lower one  $-$  is for the conformal transformed ones. Note here is required that for

$$c_1 = 1, \quad c_4 = R^3/8, \quad c_8 = 8/R^3 \quad (2.25)$$

above solutions matches with the original  $F_4^{(0)}$  in (2.6) and by conformal transformation matches with its skew-whiffed exactly.

So far, these added branes or anti-branes back-react on the original geometry although a negligible amount. However, we now see that for some other values of the constants they don't back-react never. We remember that on the RHS of the metric equation (2.10) there are energy-momentum tensors. In the case, we should try to cancel the competitor terms because of the added branes in the related tensor

$$T_{MN}^{F_4} = \frac{1}{2 \cdot 4!} \left[ 4F_{MPQR}F_N^{PQR} - \frac{1}{2}g_{MN}F_{PQRS}F^{PQRS} \right] \quad (2.26)$$

for which we just look in  $AdS_4$  components as for our forms the internal components vanish trivially. So, to nullify the back-reactions we have to set

$$c_1 = -2, \quad c_4 = -R^3/4, \quad c_8 = -4/R^3 \quad (2.27)$$

Therefore, needing no back-reaction because of the added effects on the original background, leads us to a special skew-whiffing. This in turn means that we are adding supersymmetry breaking anti D2-branes so that resultant theory is for anti-membranes. One may also note that for the conformal transformed solutions don't back-react on the main geometry, we should take the opposite value of the constants of (2.27). For both cases and all 4-forms, the

energy-momentum tensors vanishing so implies the solution we have indeed is

$$F_4 = -\frac{3}{4}R^3\mathcal{E}_4 \quad (2.28)$$

All together, one may note that we indeed have added some electric D2-branes and anti D2-branes in same directions with the fundamental near horizon branes in ABJM. Nevertheless, the interpretation may be as some fluctuations on main branes and not specially as new added branes. Anyhow, when we add D2-branes with solutions in (2.17), (2.19) and positive values of the constants we absolutely have some back-reaction even that small while with embedding the special anti-branes with solution in (2.21) and just with special negative values of the constants in (2.27) we can avoid back-reaction. We also should note that for the special values of the constants in (2.25), the solution has same structure as ABJM while with negative values there is an exactly skew-whiffed version there in (2.6). Both the later cases result in back-reaction. Therefore, we mention again that to abstain from back-reaction one may consider interactions between the main branes of ABJM with the special anti-branes added or main anti-branes with special branes added.

Therefore, we can have two bulk theories. One that preserves all supersymmetries and one that breaks all supersymmetries. The former is when same branes with ABJM are included while the latter is when one add anti-branes or some skew-whiffing of the original ones. In both cases the real scalar  $f$  here sits in  $\mathbf{1}_0$  or is a singlet of  $SU(4) \times U(1)$ . But we should note when one look at already known spectrum of the involved supergravity theories on the associated spaces [5], [6], see that the massless scalars sit in  $\mathbf{35}_v \rightarrow \mathbf{15}_0 \oplus \mathbf{10}_2 \oplus \bar{\mathbf{10}}_{-2}$  of  $SO(8) \rightarrow SU(4) \times U(1)$  for the original supersymmetric  $\mathcal{N} = 6$  theory and also for the skew-whiffing  $\mathbf{8}_c \leftrightarrow \mathbf{8}_s$  of ABJM with  $\mathcal{N} = 0$  we recently considered in [1], [4]. In other words, scalars, fermions and gravitons are originally in the representations  $\mathbf{8}_v$ ,  $\mathbf{8}_c$  and  $\mathbf{8}_s$  of  $SO(8)$  in ABJM, respectively. For now and having a  $SU(4)$ -singlet scalar in the bulk, we should swap the representations  $\mathbf{8}_s$  and  $\mathbf{8}_v$  for supercharges and scalars in ABJM while fermions remain in  $\mathbf{8}_c$  unchanged. So our new skew-whiffed representation and original ones relate as

$$\begin{aligned} \begin{cases} \mathbf{8}_v \rightarrow \mathbf{8}_s = \mathbf{1}_2 \oplus \mathbf{1}_{-2} \oplus \mathbf{6}_0 \\ \mathbf{8}_s \rightarrow \mathbf{8}_v = \mathbf{4}_1 \oplus \bar{\mathbf{4}}_{-1} \\ \mathbf{8}_c = \mathbf{4}_{-1} \oplus \bar{\mathbf{4}}_1 \end{cases} &\Rightarrow \begin{cases} \mathbf{35}_v \rightarrow \mathbf{35}_s = \mathbf{1}_0 \oplus \bar{\mathbf{1}}_4 \oplus \mathbf{1}_{-4} \oplus \bar{\mathbf{6}}_2 \oplus \mathbf{6}_{-2} \oplus \bar{\mathbf{20}}_0 \\ \mathbf{35}_s \rightarrow \mathbf{35}_v = \mathbf{10}_2 \oplus \bar{\mathbf{10}}_{-2} \oplus \mathbf{15}_0 \\ \mathbf{35}_c = \mathbf{10}_{-2} \oplus \bar{\mathbf{10}}_2 \oplus \mathbf{15}_0 \end{cases} \\ \mathbf{28}_v &\rightarrow \mathbf{1}_0 \oplus \bar{\mathbf{6}}_2 \oplus \mathbf{6}_{-2} \oplus \mathbf{15}_0 \end{aligned} \quad (2.29)$$

note that the gauge bosons sit in same representation for all three gravitinos. Now we have a uncharged  $SU(4)$ -singlet scalar ( $\mathbf{1}_0$ ) with the skew-whiffing of  $\mathbf{8}_s \leftrightarrow \mathbf{8}_v$ . It is mentionable that we indeed have a real scalar in the bulk because of its origin from a 3-form completely in the external space of  $AdS_4$ .

Here is proper to discuss on the (anti) brane charges. According to the standard formula

$$Q_e^{D2} = \frac{1}{\sqrt{2}\kappa^2} \int *F_4, \quad Q_m^{D2} = \frac{1}{\sqrt{2}\kappa^2} \int F_4 \quad (2.30)$$



where  $\kappa^2 = \frac{1}{2}(2\pi)^7$ , and making use of the following relations

$$*\mathcal{E}_4 = \frac{R^3}{3k} J^3, \quad *J^3 = \frac{k}{128R^3} \mathcal{E}_4, \quad \text{Vol}(CP^3) = \frac{\pi^3 R^9}{6k^3} \quad (2.31)$$

based on the solutions (2.17), (2.19) or (2.21), one simply obtain

$$Q_e^{D2} = \frac{4}{\sqrt{2}(2\pi)^9} \frac{C}{\lambda}, \quad Q_m^{D2} = \frac{2}{\sqrt{2}(2\pi)^7} C \quad (2.32)$$

where the full volume of  $CP^3$  and  $AdS_4$  are factored out for electric and magnetic charges, respectively and that  $C$  can be used for the overall coefficients in (2.22), (2.23) and (2.24). We see the electric charge is tiny as  $\lambda \gg 1$  and in general both charges are small compared with the background one. That is because if we use, for instance, the constants in (2.27) the resultant charge is a very small fraction of  $N$  (almost zero) justifying more that back-reaction can be ignored absolutely. Remember that plus sign is for branes and minus sing is for anti-branes.

The respective contributions from the fifth sentence of the action (2.7) based on the solutions, like charge, become

$$S_{inst.}^{D2} = -\frac{2}{(2\pi)^{11}} \frac{C^2}{\lambda^2} \quad (2.33)$$

where the full 10d volume  $\text{Vol}(AdS_4 \times CP^3)$  is factored out as a common factor. We again see that the corrections induced by new effects are really small.

## 2.3 On Ansatzs and Solutions in Eleven-Dimension

Statements in the last subsections on D2-branes are valid in case for M2-branes in general. More clearly, the ansatzs and solutions there are valid in the 11d supergravity theory over  $AdS_4 \times S^7/Z_k$  (with  $S^7/Z_k = CP^3 \times S^1/Z_k$ ) and satisfies the identity and equation

$$dF_4 = 0, \quad d *_{11} F_4 + \frac{1}{2} F_4 \wedge F_4 = 0 \quad (2.34)$$

with a note that unit-volume form in the 7d internal space is

$$\mathcal{E}_7 = \frac{1}{8 \cdot 3!} J^3 \wedge e_7, \quad e_{S^1/Z_k}^7 = \frac{1}{k} (d\varphi + k\omega) \equiv e_7 \quad (2.35)$$

which next to (2.31) are useful in evaluating the actions and charges of the added (anti) M-branes. The issue of back-reaction is also exactly same as the former (anti) D-brane case, while we don't have dilation or any more fields here. Another point to say is the ansatzs for  $F_4$  are invariant under any internal isometric symmetry and specially  $SO(8)$ , which is a special case with  $k = 1, 2$  here.

An interesting issue to address is the uplift of solution to eleven-dimension with its margins and interpretations. But before that, one may note the original solutions we have are point-

like in the external space similar to [12], [13] and not necessarily in full 10- or 11-dimensional space as [11]. Indeed the solution (2.27) and also that in [4] may be considered as smeared over  $CP^3$  or  $S^7/Z_k$ , while the original solutions (2.11), (2.12) and (2.13) are originally smeared not only on six or seven internal directions but also on three coordinates of the bulk. Namely, they are smeared in D2(M2)-branes world-volumes as well and localized just in  $u$ .

To uplift, we first note the energy-momentum tensors, because of new branes, vanish exactly when the constants are (2.27) that means adding special anti M/D-branes. This in turn means that in absence of back-reaction, our thought kaluza-klein truncation is consistent (look at [16] for instance). This point out the lower dimensional fields don't serve as sources for the upper dimensional ones. So, we try to build a full 11d solution of the 4d Laplace equation.

A solution localized in full 11d space, with all eleven indices for the Laplace equation, can be set with an ansatz as  $f(x_i, y_m) = G(u)F(\vec{u}, \vec{y})$  with  $\vec{y}$  considered as the eight coordinates transverse to M2-branes world-volume and  $u = \sqrt{y^m y^m}$ ,  $m = 1, \dots, 8$ . In same lines with [17], one can show that

$$\begin{aligned} F(\vec{u}, \vec{y}) &= c_{10} + \frac{c_{11}}{[(\vec{y} - \vec{y}_0)^2 + (\vec{u} - \vec{u}_0)^2]^3}, \quad G(u) = c_{12}u^3 \\ f(u, \vec{u}, \vec{y}) &= c_{13} + \frac{c_{14} u_0^3 u^3}{[(\vec{y} - \vec{y}_0)^2 + (\vec{u} - \vec{u}_0)^2]^3} \approx \frac{u_0^3 u^3}{|\vec{X} - \vec{X}_0|^6}, \quad \vec{X} = (x^i, y^m) \end{aligned} \quad (2.36)$$

where the eleven bosonic collective coordinates  $\vec{X}_0 \equiv (x_0^i, y_0^m)$  represent the M-instanton position in full 11d Euclidean space. This instanton solution may be a wormhole connecting the asymptotic  $AdS \times S^7/Z_k$  space and a flat space at the instanton location [18], [17]. It is also notable the solution (2.17) can be considered as  $u_0 \rightarrow \infty$  limit of (2.36) and so (2.17) suits to a large instanton. On the other hand, that in (2.21) is the  $u_0 \rightarrow 0$  limit of this 11d localized solution and so it suits to a small instanton. We return to this point soon.

## 3 On Field Theory Side Aspects

### 3.1 M2/D2-Branes Standard Lagrangian

We just mention the explicit  $SU(4)$  invariant lagrangian of the ABJM [5] model, which is at hand here. For M2-branes of 11d supergravity the near horizon geometry is  $AdS_4 \times S^7$ . Because of ABJM, when  $N$  stacks of these branes probe a  $C^4/Z_k$  singularity, the world-volume theory of branes is a  $\mathcal{N} = 6$  conformal Chern-Simon matter field theory with a quiver gauge group of  $U(N)_k \times U(N)_{-k}$ . The matter fields transform in the bi-fundamental representations of the gauge group with the Chern levels of  $(k, -k)$ . In a special limit when the 't Hooft boundary effective coupling of  $\lambda \equiv N/k \gg 1$  and  $k^5 \gg N$ , a better description of gravity theory is type IIA supergravity over  $AdS_4 \times CP^3$ . In the procedure,  $S^7$  is always considered as a  $U(1)$  fibration on  $CP^3$ . On the field theory side, this  $U(1)$  symmetry matches to  $U(1)_b$  with  $b$  for baryonic symmetry.

The  $SU(4)_R \times U(1)_b$  invariant standard action for M2/D2-brane theories now is

$$S_{ABJM} = \int d^3x \left\{ \frac{k}{4\pi} \varepsilon^{\mu\nu\lambda} \text{tr} \left( A_\mu A_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \right) \right. \\ \left. - \text{tr} (D_\mu Y_A^\dagger D^\mu Y^A) - \text{tr} (\psi^{A\dagger} i \gamma^\mu D_\mu \psi_A) - V_{bos} - V_{ferm} \right\} \quad (3.1)$$

where

$$V_{ferm} = -\frac{2\pi i}{k} \text{tr} (Y_A^\dagger Y^A \psi^{B\dagger} \psi_B - Y^A Y_A^\dagger \psi_B \psi^{B\dagger} + 2Y^A Y_B^\dagger \psi_A \psi^{B\dagger} - 2Y_A^\dagger Y^B \psi^{A\dagger} \psi_B \\ + \varepsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D - \varepsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger}) \quad (3.2)$$

$$V_{bos} = -\frac{4\pi^2}{3k^2} \text{tr} (Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger \\ - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger) \quad (3.3)$$

are the Bose-Fermi interaction term and scalar bosonic potential, respectively- note here that  $\mu, \nu, \dots$  stand for 3d Minkowski indices. The matter fields are four complex scalars  $Y^A$  ( $A = 1, 2, 3, 4$ ) and four three-dimensional spinor  $\psi_A$ , which transform in the bi-fundamental representation  $(\mathbf{4}_1, \bar{\mathbf{4}}_{-1})$  of  $SU(4)_R \times U(1)_b$ . The gauge fields  $A_\mu$  and  $\hat{A}_\mu$  couple to the matter fields  $\Phi$  ( $Y^A$  or  $\psi_A$ ) by means of the covariant derivatives

$$D_\mu \Phi = \partial_\mu \Phi + i A_\mu \Phi - i \Phi \hat{A}_\mu, \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu, A_\nu] \quad (3.4)$$

where the field-strength for  $A_\nu$  is also given. The conventions for metric, Clifford algebra and real gamma matrices in original Minkowski signature read

$$\eta_{\mu\nu} = \text{diag}(-1, 1, 1), \quad \{\gamma_\mu, \gamma_\nu\} = -2\eta_{\mu\nu}, \quad \gamma^\mu = (i\sigma_2, \sigma_1, \sigma_3), \quad \varepsilon_{012} = -1 \quad (3.5)$$

where  $\sigma_{1,2,3}$  are usual Pauli matrices. It is also mentionable that traces take on  $N \times N$  matrices of the gauge group keeping the gauge invariant quantities and that normalization for the  $U(N)$  generators of  $t^a$  is set as  $\text{tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$ .

This lagrangian was first studied in [5], [7] whilst the  $\mathcal{N} = 1, 2$  superfield formalism of theory was presented in [19], [7] next to some other related aspects that were also surveyed in [20], [21]. In [22], it was shown that for the special case of  $N = 2$  M2-branes, the Bagger-Lambert-Gustavsson (BLG) theory is a special case of ABJM with the gauge group of  $SU(2) \times SU(2)$ . Indeed, for the special cases  $k = 1, 2$ , the  $SU(4)$  R-symmetry of ABJM is enhanced to  $SO(8)$  and therefore  $\mathcal{N} = 8$  owing to monopole operators [23].

## 3.2 Matching the Bulk Solutions to the Boundary

For a scalar field in bulk of  $AdS_4$  when approaching the boundary at  $u = 0$  of the Poincare upper-half plane coordinates (2.3), behaves like [8]

$$f(u, \vec{u}) \approx \alpha(\vec{u}) u^{\Delta_-} + \beta(\vec{u}) u^{\Delta_+} \quad (3.6)$$

where  $\Delta_{\mp}$  are the roots of  $(mL)^2 = \Delta(\Delta - 3)$ . For a massless scalar,  $\Delta_{\mp} = 0, 3$  from which we use  $\Delta_+ = 3$  that corresponds to the normalizable mode in the bulk.  $\alpha$  and  $\beta$  play roles as *source* and *vacuum expectation value* of a marginal operator of the conformal dimension of  $\Delta_+ = 3$ , respectively and vice versa for  $\Delta_-$  [8], [24].

Such a scalar can be quantized either with Dirichlet boundary condition  $\delta\alpha = 0$  (which can use for any  $m^2$ ) or with Neumann or mixed boundary condition  $\delta\beta = 0$  (which can use when the scalar masses are in range of  $-9/4 < m^2 L^2 < -5/4$  ensuring stability too). For the Dirichlet boundary condition, stability requires the scalar mass obeys Breitenlohner-Freedman (BF) bound  $m^2 L^2 \geq -9/4$  [25]. These boundary conditions preserve the asymptotic symmetry groups, lead to finite energy and correspond to two boundary CFT's. The *usual* CFT is that for which a source  $\alpha$  couples to an operator of the dimension  $\Delta_+$ . Anyway, we now note that for the solution in (2.17) both  $\alpha$  and  $\beta$  are constants, for which we will propose an operator of the conformal dimension of 3. Meanwhile, for the solution of (2.21) the procedure here is same as that outlined in [4]. Indeed, comparing (2.21) with (3.6) we can write

$$\alpha(\vec{u}) = f_0(\vec{u}), \quad \beta(\vec{u}) = \frac{c_2}{|\vec{u} - \vec{u}_0|^6} \quad (3.7)$$

for a localized object on the boundary the source of  $f_0(\vec{u}_0)$  is indeed a delta function  $\delta^3(\vec{u} - \vec{u}_0)$  and we have

$$\frac{1}{3} \langle \mathcal{O}_3(\vec{u}) \rangle_{\alpha} = -\frac{\delta W[\alpha]}{\delta \alpha} = \beta(\vec{u}) \quad (3.8)$$

in which  $W = -S_{on-shell}$  are the field theory generating functional and the bulk on-shell action we evaluate bellow. Then in language of [9], because of turning on the normalizable bulk scalar mode we should deform the action as  $S \rightarrow S + W$ , where

$$W = -\frac{1}{3} \int d^3 \vec{u} \alpha(\vec{u}) \mathcal{O}_3(\vec{u}) \quad (3.9)$$

with noting that for our solutions  $\alpha = c_1$  and the plain forms of operators come soon.

## 3.3 Boundary Solutions and Correspondence

First we remember that our ansatzs for  $F_4$  are  $SU(4)$  and  $SO(8)$  invariant and actually singlet, in addition they don't carry any  $U(1)$  charge. So, the dual operator  $\mathcal{O}_3$  should have same property. Nevertheless, we already know the normalizable mode may be considered as a different state in the same theory and not necessarily as a deformation of that [24]. This fact suggests the dual operators have same structures as the main lagrangian (3.1) terms. This

statement is confirmed in some previous studies on the spectrums and Bogomol'nyi-Prasad-Sommerfeld (BPS) operators to which the bulk modes agree [26], [27], [30],[29]. The common proposed marginal operator dual to bulk scalars can read

$$\mathcal{O}_3 = \text{tr}(X^{[I} X_J^\dagger X^{K]} X_{[K}^\dagger X^J X_{I]}^\dagger) \quad (3.10)$$

where suitable trace subtractions is mentionable. This operator has of course same structure with the scalar sextet potential in BLG and ABJM model as if one use  $Y^A = X^A + iX^{A+4}$ , the ABJM scalar potential (3.3) coincide with BLG one, which is  $-\frac{32\pi^2}{3k^2}\mathcal{O}_3$  [22], [7].

On the other hand, we recall that we have two types of solutions. One may attribute to adding some D/M-branes in same directions with the original ones in near horizon of ABJM and thus preserving all symmetries and supersymmetry. While the other one may attribute to adding some anti D/M-branes with a flipped direction with respect to the original ones because of the conformal transformation (2.20), again preserving all symmetries but breaking all supersymmetries. Now for both cases we try to build solutions.

By the way, we know that massless scalars in original ABJM theory sit in the representation  $\mathbf{35}_v \rightarrow \mathbf{15}_0 \oplus \mathbf{10}_2 \oplus \bar{\mathbf{10}}_{-2}$  of  $SO(8) \rightarrow SU(4) \times U(1)$  with  $X^I \rightarrow (Y^A, Y_A^\dagger)$ . This marks the  $U(1)$ -natural ones are in  $\mathbf{15}_0$  while our bulk scalars are singlet;  $\mathbf{1}_0$ . But no problem; because the representation for  $Y^A Y_A^\dagger$  is  $\mathbf{4}_1 \otimes \bar{\mathbf{4}}_{-1} = \mathbf{15}_0 + \mathbf{1}_0$  and so from (3.10) one can easily see there is a  $SU(4)_R \times U(1)_{b-}$  singlet. Therefore, with a more intimately 3d operator composed of the ABJM scalars like

$$\mathcal{O}_3 = \text{tr}(Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger) \quad (3.11)$$

to agree with a bulk solution like (2.17) with respect to (3.8) and (3.9), one may simply set

$$Y^A = Y_A^\dagger = c_{15} \mathbf{I}_{N \times N} \quad (3.12)$$

where  $\mathbf{I}_{N \times N}$  is the unitary or identity matrix meanwhile no striking point seems to appear for the case. But for the conformally transformed or skew-whiffed solution of (2.21), which corresponds to anti M/D-brane theories, the situation is intersecting.

As discussed in (2.29), for the skew-whiffed bulk solution, we should exchange the representations  $\mathbf{s}$  and  $\mathbf{v}$  for supercharges and scalars of ABJM so now scalars sit in  $\mathbf{8}_v \rightarrow \mathbf{8}_s = \mathbf{1}_2 \oplus \mathbf{1}_{-2} \oplus \mathbf{6}_0$ , according to which we break up the scalar fields as  $X^I \equiv (y^n, y^7, y^8) \approx (y^n, y, \bar{y})$  with  $n = 1, \dots, 6$ ,  $y = y^7 + iy^8$  and for simplicity  $\bar{y} = y^\dagger$ . The situation is similar with [1], [4], where we had a single fermion instead of the scalar here. Making use of just this single scalar along with setting other six scalars to zero, the marginal operator from (3.11) reads

$$\mathcal{O}_3 = \text{tr}(y\bar{y})^3 \quad (3.13)$$

meanwhile there are still other ways to build such an operator among scalars. It is simple to see that with  $y^n = 0$  the scalar potential from the action of (3.1) vanish. Afterwards, by setting the gauge fields and also fermions to zero and with respect to (3.9), the remaining part

of lagrangian reads <sup>1</sup>

$$\mathcal{L}_{inst.} = -tr(\partial_i y \partial_i \bar{y}) - \frac{c_1}{3} tr(y \bar{y})^3 \quad (3.14)$$

next taking  $y = ih(r)\mathbf{1}_{1 \times 1}$ , we arrive in

$$h(r) = \sqrt[1/2]{\frac{3}{c_1} \left( \frac{\dot{c}}{\dot{c}^2 + r^2} \right)}^{1/2} \quad (3.15)$$

with  $\dot{c}$  another constant. Then based on the current solution with  $c_1 = 1$  the finite part of the action becomes

$$S_{inst.S} = \frac{\sqrt{3}}{2} \pi^2 \quad (3.16)$$

Now as the main test for the correctness of duality, we see the one-point function of  $\mathcal{O}_3(\vec{u})$  based on the solution is same as  $\beta(\vec{u})$  in (3.7) up to some constants, which adjust together.

To sum up the subsection, we remind that our bulk solutions with  $\mathcal{N} = 0, 6$  supersymmetry have some equivalents in the ABJM  $SU(4)_R \times U(1)_b$ -invariant action especially for  $k \geq 3$  when we use the Hopf-fibred  $S^7/Z_k = CP^3 \times S^1/Z_k$  in correspondence with already known spectrums [30]. For  $k = 1, 2$ , our bulk solution is tantamount to the boundary theory with  $SO(8)$  R-symmetry and so  $\mathcal{N} = 8$  supersymmetry. The crucial point is that as long as the ABJM scalar potential we use to match solutions is  $SO(8)$ -invariant [23], [22], we can say implicitly the solutions here are valid for all  $k$ 's; although in general  $SO(8)$  symmetry combines the standard operators like  $tr(Y^A Y_A^\dagger)^\ell$  with monopole-operators to enhance the symmetry [5].

Nonetheless, we may still go another way for matching the bulk to boundary solutions. That is using other common terms in BLG and ABJM theories. So, we use the Chern-Simon terms in the next section. We will see that using the gauge fields is meaningful both in finding the boundary solutions for  $k = 1, 2$  that were obscured in [4] too and also following the lines already used for finding D-instantons in 10d type IIB theory over  $AdS_5 \times S^5$  versus 4d  $SU(N)$   $\mathcal{N} = 4$  Yang-Mills field theory.

## 4 The Bulk Solution from Chern-Simon Action

We now consider just the universal Chern-Simon terms and gauge fields of the action (3.1) for M2/D2-branes. For convenience and sake of solution matchings, we put away one group of the full  $U(N)_k \times U(N)_{-k}$ ; that means then using one of the gauge fields, say  $A_i$ . Still an even more logical reason doing so may come from a *novel Higgs mechanism*, where the quiver gauge group  $SU(N) \times SU(N)$  breaks down to  $SU(N)$  under certain conditions- look at [31] and references therein for instance. Another reason could be parity breaking by the supersymmetry breaking solution of [4] on related situations some typical studies are also done in [32]. Anyhow, we simply take  $\hat{A}_i = -A_i$ . So, the remaining lagrangian with setting scalars

---

<sup>1</sup>It is notable that one could simply set all fields in the action of (3.1) to zero except one real scalar field, and then get the wished solution trivially without dealing with any deformation as the marginal normalizable bulk mode suggest the boundary solution may be from the main theory and not any deformation of that.

and fermions to zero reads

$$\mathcal{L}_{CS} = \frac{k}{4\pi} \text{tr} \left( A \wedge dA + \frac{2i}{3} A \wedge A \wedge A \right), \quad F = dA + iA \wedge A \quad (4.1)$$

On the other hand, we know that to have a finite action by a configuration, the lagrangian density should be nonzero just in a localized region of space and vanish at boundary of the Euclidean space. It means that for the condition  $F_{ij} = 0$  here,  $A_i$  should behaves as a pure gauge at infinity or origin. However, it is well-known that all gauge fields with vanishing field-strengths at infinity can be classified by an integer number  $K$  named as *instanton number* or *Pontryagin index* [33]

$$K = \frac{1}{16\pi^2} \int_{R^4} d^4x \text{tr}(F_{\dot{\mu}\dot{\nu}} *_4 F^{\dot{\mu}\dot{\nu}}), \quad *_4 F_{\dot{\mu}\dot{\nu}} = \frac{1}{2} \mathcal{E}_{\dot{\mu}\dot{\nu}\dot{\rho}\dot{\sigma}} F^{\dot{\rho}\dot{\sigma}} \quad (4.2)$$

with noting of the conformal flatness of Euclidean  $AdS_4$ . By using the Stocks-theorem, the integral on  $R^4$  can be reduced to an integral on  $\partial R^4 \approx S^3$ . So, for our interest we have

$$K = \frac{1}{8\pi^2} \int_{R^3 \approx S^3} d^3x \varepsilon^{ijk} \text{tr} \left( A_i \partial_j A_k + \frac{2i}{3} A_i A_j A_k \right) \quad (4.3)$$

here is same structure we deal with in the action. Now to adjust with bulk solution in (2.21) and [4], we use the special *singular* gauge of

$$A_i = \frac{u^2}{r^2 + u^2} g^{-1} \partial_i g, \quad g = \frac{(u1_2 - ix_i \sigma^i)}{(r^2 + u^2)^{1/2}} \quad (4.4)$$

where the gauge field  $A_i$  behaves as a pure gauge on the boundary ( $\vec{u} \rightarrow \vec{u}_0$ ) and  $g$  is a element of  $SU(N)$ . We use here the simplest case  $SU(2)$  of course. This simple case is even more relevant in that with  $k = 1, 2$  and  $SO(8)$  R-symmetry, the gauge group of BLG is indeed  $SU(2) \times SU(2)$  [22], [7], which is in turn the case with two M2-branes in ABJM [5].

Now by evaluating the respective part of the action in (3.1) with hinting that it acquires a  $i$  factor because of Wick-rotation to the Euclidean space, we have

$$S_{CS} = \frac{k}{6\pi} \int_{S^3} \text{tr}(A^3), \quad S_{inst.YM} = 4\pi k \quad (4.5)$$

where we have taken the integral on the north pole ( $u = 1, \vec{u} = 0$ ) of unit-sphere with noting that  $F = 0$  for pure gauge. We see this is same value with that got in [4] by another way and also with that in (3.1) nearly. Note also the topological charge here is  $K = -1$ .

As an other side, we try to follow similar lines with [13], [17], where instanton was indeed a D(-1)-brane inside N background D3-brane world-volumes. To do so, we note the instanton charge we have because of adding some anti D/M-branes is

$$Q_{inst.} = \int_{AdS_4 \times CP^3 | S^7/Z_k} *(d * df) \quad (4.6)$$

where  $f$  is that in (2.21) and Hedge-star operation just here is in 10- or 11-dimension. By noting the last integrand is indeed Laplace equation, we have

$$Q_{inst.} = \frac{c_2}{3Vol(\partial AdS_4)} \lim_{u \rightarrow 0} \int_{R^3} d^3x \frac{1}{u^3} \frac{u^3}{(u^2 + r^2)^3} = \frac{c_2}{3} \quad (4.7)$$

finally where the internal space volume is factored out.

On the other hand, we recall the conformal transformation (2.20) allows us to relate the behavior of system at origin and at infinity, where the associated asymptotic spaces may connect by a throat. In other words, the original solution (2.17) for branes is singular at infinity in contrast to its conformally transformed (2.21) for anti-branes, which is singular at  $u = 0$ . The latter corresponds to a small instanton on the boundary that leaves the background geometry unchanged. Similar to D-instanton in type IIB over  $AdS_5 \times S^5$  in string frame, the configuration is composed of two asymptotic  $AdS_4 \times S^7$  ( $CP^3$ ) spaces one at singularity at origin and another at infinity, which are connected by a throat [18], [17]. One interpretation may be that we here have branes at infinity and anti-branes on the boundary, which are connected by a throat from which the instanton charge flows. With this language, the instanton number  $K$  in (4.3) may be identified with the instanton charge  $Q_{inst.}$  as

$$K \approx Q_{inst.} \approx \lim_{u \rightarrow 0} \int_{R^3} d^3x \frac{1}{u^3} \frac{u^3}{(u^2 + r^2)^3} \quad (4.8)$$

It is mentionable the solution of (2.21) matches to a small Yang-Mills instanton when we approach the boundary at  $u = 0$ . In other words, three coordinates of  $\vec{u}_0$  describe the instanton location while the scalar parameter  $u_0$  is for the instanton size. They are indeed four moduli space parameters of  $SO(4, 1)/SO(4)$ , where  $SO(4, 1)$  is the isometry group of Euclidean  $AdS_4$  and  $SO(5)$  is a subgroup consists of rotations, translations and special conformal transformations keeping invariant instanton up to some gauge transformations. We should also note that one can gain  $SU(N)$  instantons by some embedding of  $SU(2)$  instantons into  $SU(N)$  so that various embeddings lead to various configurations [34].

Further, we note that because of the presence of D-instanton on D3-branes, the gauge field  $F$  on D3-brane world-volume couples to the axion field of  $A_0$  with  $F_1 = dA_0$  as  $S_{WZ} \sim \int A_0 F \wedge F$  [12], [13], [11]. What we can say here? We may roughly name the object some M- or D-instanton; but a more probable counterpart may be  $F_0 = df$  as  $S_{WZ} \sim \int F_0 A \wedge A$ . Indeed by taking 10- or 11-dimensional Hodge-star on the latter 1-form and then taking exterior derivative, the  $AdS_4$  part of the resultant 10-form is the integrand in (4.6). In a known language, this 0-form is named as *Romans mass*, which play here similar role as if was a D(-2)-brane in type IIA like D(-1)-brane in type IIB. Indeed it was already shown in [35] there are such  $\mathcal{N} = 0$  supersymmetry breaking solutions of ABJM.



## 5 Summary and Comments

This work is on trail of recent studies [1], [2], [3], [4] in searching for vacua and mainly instantons of M2/D2-brane theory in framework of  $AdS_4/CFT_3$  correspondence. We applied some 4-form field-strengths of 10- and 11-dimensional supergravities in terms of completely  $AdS_4$  ingredients to stand some solutions in the bulk. Actually when the EOM's apply to ansatzs, we have a solution just localized in the horizon coordinate of  $u$ , which is of course associated with some special brane adding on same directions with the background ones. As the solutions are invariant under a special conformal transformation we then gain a special skew-whiffed solution localized completely in Euclidean  $AdS_4$  avoiding back-reaction. This solution-also in [4]- is associated with branes with flipped directions with respect to the main D2/M2-branes in near horizon of ABJM model [5] that breaks all supersymmetries while preserving other symmetries of the original theory. Because the bulk normalizable mode is a  $SU(4)$  and  $SO(8)$ -singlet massless scalar natural with respect to  $U(1)$ , the dual boundary operator with conformal dimension of  $\Delta_+ = 3$  should be also same singlet. On the other hand, from known spectrums of the gauged supergravities on the associated spaces  $AdS_4 \times S^7/Z_k|CP^3$  [6], one see when the gravitinos are in  $\mathbf{8}_{s,c}$ , the uncharged scalars are in  $\mathbf{15}_0$  of  $SU(4)$ ; while for the gravitino  $\mathbf{8}_v$  they are in both  $\mathbf{1}_0$  and  $\mathbf{20}_0$ . However, this is invalid for  $k = 1, 2$  of the quotient space  $S^7/Z_k$  [30]. The form of such marginal operators are of same type with sentences in the main ABJM lagrangian (3.1) [24] conjectured also in [26], [27], [28]. For the current case, they are in turn of same structure with the bosonic potential of (3.3). For the thought added branes, the operator is trivial meantime for anti-branes we should swap the representations  $\mathbf{8}_s$  and  $\mathbf{8}_v$  for supercharges and scalars in ABJM while the fermion remain fixed in  $\mathbf{8}_c$ . After that, we used a single scalar to built the dual operator and then by making use of known AdS/CFT duality rules [8], [9], we matched the bulk solution to a clear boundary one found. In addition, we used another possibility to match with the fully localized bulk object, suitable for  $k = 1, 2$  cases particular. That is the Yang-Mills fields of the common Chern-Simon terms from the standard M2/D2-brane lagrangian [22], [7], [5]. With keeping just one of the gauge group and then a  $SU(2)$  subgroup, we found a dual boundary solution equivalent to the instanton of  $AdS_4/CFT_3$  duality [11], [12], [17], [13] where a Rommans mass  $F_0$  here may play a similar role as the axion  $A_0$  there.

Another point to say is about the magnetic dual of these new D2- and M2-(anti) branes. Indeed, we note the magnetic duals of the 4-forms here couple to D4-(anti)branes and M5-(anti)branes, respectively. Then, the world-volume of such (anti)branes supposed to wrap around some five and six directions of  $J^3$  and  $J^3 \wedge e_7$ , respectively. That is a similar situation already encountered in [4], where anti D4-brane supposed to wind around  $J^2 \wedge \omega$  and its uplifted anti M5-brane in 11d supposed to wind around  $J^2 \wedge \omega \wedge d\varphi$ .

Finally, we saw for the special combination of the constants in solutions the back-reactions are vanished and so we argue the bulk scalar don't disturb the upper dimensional fields as a truncation supposed now to be consistent [16], [14]. However, although the supersymmetry always ensures stability and the skew-whiffed solution is stable just for  $S^7$ , for similar situations here the stability is already established [25].

## 6 Acknowledgements

The world of somebodies is smaller than a village among a world of almost 170 billion galaxies in the observable universe. Hitherto no problem in that is their fault! But when they come to power and impose their brilliant perceptions and paralyze many life and humanitarian standards, it is an unforgivable sin. Therefore, it seems that I/we should really pray! to the peoples who have governed their limited thoughts frozen in that special village over many of other peoples who maybe know there are many other worlds among them we are probably nothing! So, I absolutely say shame on all small and big dictators who govern and decide for peoples except based on the accepted international rules.

## References

- [1] A. Imaanpur and M. Naghdi, "*Dual Instantons in Anti-membranes theory*", Phys. Rev. D 83, 085025 (2011), [arXiv:1012.2554 [hep-th]].
- [2] M. Naghdi, "*A monopole Instanton-like effect in the ABJM model*", Int. J. Mod. Phys. A 26, 3259 (2011), [arXiv:1106.0907 [hep-th]].
- [3] A. Imaanpur, " *$U(1)$  Instantons on  $AdS_4$  and the uplift to exact supergravity solutions*", JHEP 1111, 041 (2011), [arXiv:1108.2786 [hep-th]].
- [4] M. Naghdi, "*New Instantons in  $AdS_4/CFT_3$  from  $D4$ -Branes Wrapping Some of  $CP^3$* ", [arXiv:1302.5294 [hep-th]].
- [5] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, " *$\mathcal{N}=6$  superconformal Chern-Simons matter theories,  $M2$ -branes and their gravity duals*", JHEP 0810, 091 (2008), [arXiv:0806.1218 [hep-th]].
- [6] B. E. W. Nilsson and C. N. Pope, "*Hopf fibration of eleven-dimensional supergravity*", Class. Quant. Grav. 1, 499 (1984).
- [7] M. Benna, I. Klebanov, T. Klose and M. Smedback, "*Superconformal Chern-Simons theories and  $AdS_4/CFT_3$  correspondence*", JHEP 0809, 072 (2008), [arXiv:0806.1519].
- [8] I. R. Klebanov and E. Witten, " *$AdS/CFT$  correspondence and symmetry breaking*", Nucl. Phys. B 556, 89 (1999), [arXiv:hep-th/9905104].
- [9] E. Witten, "*Multi-trace operators, boundary conditions, and  $AdS/CFT$  correspondence*", [arXiv:hep-th/0112258].
- [10] M. B. Green and M. Gutperle, "*Effects of  $D$ -instantons*", Nucl. Phys. B 498, 195 (1997), [arXiv:hep-th/9701093].

- [11] M. Bianchi, M. Green, S. Kovacs and G. Rossi, "*Instantons in supersymmetric Yang-Mills and D-instantons in IIB superstring theory*", JHEP 9808, 013 (1998), [arXiv:hep-th/9807033].
- [12] C. S. Chu, P. M. Ho and Y. Y. Wu, "*D-instantons in  $AdS_5$  and instantons in  $SYM_4$* ", Nucl. Phys. B 541, 179 (1999), [arXiv:hep-th/9806103].
- [13] I. I. Kogan and G. Luzón, "*D-instantons on the boundary*", Nucl. Phys. B 539, 121 (1999), [arXiv:hep-th/9806197].
- [14] S. de Haro, S. N. Solodukhin and K. Skenderis, "*Holographic reconstruction of spacetime and renormalization in the  $AdS/CFT$  correspondence*", Commun. Math. Phys. 217, 595 (2001), [arXiv:hep-th/0002230].
- [15] H. Liu and A. A. Tseytlin, " *$D3$ -brane- $D$ -instanton configuration and  $\mathcal{N}=4$  super YM theory in constant self-dual background*", Nucl. Phys. B 553, 231 (1999), [arXiv:hep-th/9903091].
- [16] J. P. Gauntlett and O. Varela, "*Consistent Kaluza-Klein reductions for general supersymmetric  $AdS$  solutions*", Phys. Rev. D 76, 126007 (2007), [arXiv:0707.2315 [hep-th]].
- [17] C. Park and Sand-J Sin, "*Notes on  $D$ -instantons corrections to  $AdS_5 \times S^5$  geometry*", Phys. Lett. B 444, 156 (1998), [arXiv:hep-th/9807156].
- [18] E. Bergshoeff and K. Behrndt, "*D-instantons and asymptotic geometries*", Class. Quant. Grav. 15, 1801 (1998), [arXiv:hep-th/9803090].
- [19] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, "*A massive study of  $M2$ -brane proposals*", JHEP 0809, 113 (2008), [arXiv:0807.1074 [hep-th]].
- [20] S. Terashima, "*On  $M5$ -branes in  $\mathcal{N} = 6$  membrane action*", JHEP 0808, 080 (2008), [arXiv:0807.0197 [hep-th]].
- [21] K. Hanaki and H. Lin, " *$M2$ - $M5$  systems in  $\mathcal{N}=6$  Chern-Simons theory*", JHEP 0809, 067 (2008), [arXiv:0807.2074 [hep-th]].
- [22] M. Van Raamsdonk, "*Comments on the Bagger-Lambert theory and multiple  $M2$ -brane*", JHEP 0805, 105 (2008), [arXiv:0803.3803 [hep-th]].
- [23] A. Gustavsson and S. J. Rey, "*Enhanced  $\mathcal{N}=8$  Supersymmetry of ABJM Theory on  $R^8$  and  $R^8/Z_2$* ", [arXiv:0906.3568 [hep-th]].
- [24] V. Balasubramanian, P. Kraus and A. Lawrence, "*Bulk vs boundary dynamics in anti-de Sitter spacetime*", Phys. Rev. D 59, 046003 (1999), [arXiv:hep-th/9805171].
- [25] P. Breitenlohner and D. Z. Freedman, "*Stability in gauged extended supergravity*", Annals Phys. 144, 249 (1982).

- [26] O. Aharony, Y. Oz and Z. Yin, "*M-theory on  $AdS_p \times S_{11-p}$  and superconformal field theories*", Phys. Lett. B 430, 87 (1998), [arXiv:hep-th/9803051].
- [27] S. Minwalla, "*Particles on  $AdS_{4/7}$  and primary operators on  $M_{2/5}$  brane worldvolumes*", JHEP 9810, 002 (1998), [arXiv:hep-th/9803053].
- [28] E. Halyo, "*Supergravity on  $AdS_{4/7} \times S_{7/4}$  and M-branes*", JHEP 9804, 011 (1998), [arXiv:hep-th/9803077].
- [29] E. D'Hoker and B. Pioline, "*Near-extremal correlators and generalized consistent truncation for  $AdS_{4|7} \times S_{7|4}$* ", JHEP 0007, 021 (2000), [arXiv:hep-th/0006103].
- [30] E. Halyo, "*Supergravity on  $AdS_{5/4} \times$  Hopf fibrations and conformal field theories*", Mod. Phys. Lett. A 15, 397 (2000), [arXiv:hep-th/9803193].
- [31] X. Chu, H. Nastase, B. Nilsson and C. Papageorgakis, "*Higgsing  $M2$  to  $D2$  with gravity:  $\mathcal{N}=6$  chiral supergravity from topologically gauged ABJM theory,*", JHEP 1104, 040 (2011), [arXiv:1012.5969 [hep-th]].
- [32] M. Fujita, W. Li, S. Ryu, and T. Takayanagi, "*Fractional Quantum Hall Effect via holography: Chern- Simons, edge states, and hierarchy*", JHEP 0906, 066 (2009), [arXiv:0901.0924 [hep-th]].
- [33] A. A. Belavin, A. M. Polyakov, A. S. Shvarts and Yu. S. Tyupkin, "*Pseudoparticle solutions of the Yang-Mills equations*", Phys. Lett. B 59, 85 (1975).
- [34] A. V. Belitsky, S. Vandoren and P. van Nieuwenhuizen, "*Yang-Mills and D-instantons,*", Class. Quant. Grav. 17, 3521 (2000), [arXiv:hep-th/0004186].
- [35] D. Gaiotto and A. Tomasiello, "*The gauge dual of Romans mass*", JHEP 1001, 015 (2010), [arXiv:0901.0969 [hep-th]].